First Semester Examination
2017/2018 Academic Session

January 2018

MAT518 - Numerical Methods for Differential Equations
[Kaedah Berangka untuk Persamaan Pembezaan]

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **FIVE (5)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA (5)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **all four (4)** questions.

*[**Arahan :** Jawab **semua empat (4)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

Consider $U_t = U_{xx}$

(i) Write down the Forward Time Centred Space (FTCS) scheme.

(ii) Compute the truncation error of the scheme.

(iii) Taking (ii) into account, investigate the consistency of the scheme.

[100 marks]

Soalan 1

Pertimbangkan $U_t = U_{xx}$

(i) Tulis skema masa kedepan, ruang ke pusat.

(ii) Kira ralat pangkasan skema.

(iii) Dengan mengambilkira (ii), kaji kekonsistenan skema.

[100 markah]

Question 2

Consider $T_t + uT_x = 0$ where u is a known constant.

(i) By considering the Taylor expansion for $T(x_i, t_n + \Delta t)$, derive the Lax-Wendroff relation.

$$T_t = \frac{T_i^{n+1} - T_i^n}{\Delta t} - 0.5\Delta t u^2 T_{xx}$$

(ii) Derive the Lax-Wendroff scheme for $T_t + uT_x = 0$. Recall that the Lax-Wendroff scheme uses the Lax-Wendroff relation for T_t and central difference for T_x.

[120 marks]

Soalan 2

Pertimbangkan $T_t + uT_x = 0$ *yang mana* u *ialah suatu pemalar yang diketahui.*

(i) Dengan mempertimbangkan pengembangan Taylor untuk $T\left(x_i, t_n + \Delta t\right)$*, terbitkan hubungan Lax-Wendroff.*

$$T_t = \frac{T_i^{n+1} - T_i^n}{\Delta t} - 0.5\Delta t u^2 T_{xx}$$

(ii) Terbitkan skema Lax-Wendroff untuk $T_t + uT_x = 0$*. Ambil kira skema Lax-Wendroff menggunakan hubungan Lax-Wendroff untuk* T_t *dan beza pusat untuk* T_x *.*

[120 markah]

Question 3

(i) Consider Laplaces equation

$$u_{xx} + u_{yy} = 0$$

with $R = \{(x, y) \mid 0 < x < 1, 0 < y < 1\}$ and

boundary conditions $\begin{cases} u(x,0) = 0, 0 \le x \le 1 \\ u(x,1) = \sin \pi x, 0 \le x \le 1 \\ u(0,y) = 0, 0 \le y \le 1 \\ u(1,y) = 0, 0 \le y \le 1 \end{cases}$

If n = m = 4, set up (but do not solve) the associated linear system which when solved will give *u* at the grid points. Will the solution change with time?

(ii) Consider the system

$$\begin{aligned} 3x + y - z &= 4 \\ 2x + 4y + z &= 1 \\ -x + 2y + 5z &= 1 \end{aligned}$$

Carry out 3 iterations of the Jacobi method and 3 iterations of Gauss-Seidel method.

[80 marks]

Soalan 3

(i) *Pertimbangkan persamaan Laplace*

$$u_{xx}+u_{yy}=0$$

dengan $R=\{(x,y)\mid\ 0<x<1,0<y<1\}$ *dan*

Syarat sempadan $\begin{cases} u(x,0)=0,0\le x\le 1 \\ u(x,1)=\sin\pi x,0\le x\le 1 \\ u(0,y)=0,0\le y\le 1 \\ u(1,y)=0,0\le y\le 1 \end{cases}$

Jika n=m=4, bentuk sistem linear bersekutu (tetapi jangan selesaikan) yang apabila diselesaikan akan memberikan u di titik-titik grid. Adakah penyelesaian akan berubah dengan masa?

(ii) *Pertimbangkan sistem*

$$\begin{aligned} 3x+y-z&=4 \\ 2x+4y+z&=1 \\ -x+2y+5z&=1 \end{aligned}$$

Lakukan 3 lelaran menggunakan kaedah Jacobi dan 3 lelaran menggunakan kaedah Gauss-Seidel.

[80 markah]

Question 4

(a) Consider $T_t+uT_x=0$ with u being a known constant.

(i) Write down the Leapfrog scheme i.e centred time, centred space

(ii) Obtain the amplification factor (i.e. λ) associated with a Fourier (Von Neumann) stability analysis.

(b) Write notes (approximately 1 page) on the shooting method for

$$y''=f(x,y,y')$$
$$y(a)=A,y(b)=B$$

[100 marks]

Soalan 4

(a) *Pertimbangkan* $T_t + uT_x = 0$ *dengan* u *suatu pemalar yang diketahui nilainya.*

(i) *Tuliskan skema lompat katak iaitu beza masa, beza pusat.*

(ii) *Dapatkan faktor amplifikasi (iaitu* λ*) yang bersekutu dengan analisis Kestabilan Fourier (Von Neumann).*

(b) *Tulis nota (lebih kurang 1 muka surat) berkaitan kaedah tembak untuk*

$$y'' = f(x, y, y')$$
$$y(a) = A, y(b) = B$$

[100 markah]

- ooo00ooo -